International Journal of Applied Mathematics & Statistical Sciences (IJAMSS)
ISSN(P): 2319-3972; ISSN(E): 2319-3980
Vol. 3, Issue 6, Nov 2014, 45-56

International Academy of Science, Engineering and Technology
Connecting Researchers; Nurturing Innovations

NEW ADJUSTED BIASED REGRESSION ESTIMATORS BASED ON SIGNAL-TO-NOISE RATIO

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ABSTRACT

In this paper, new adjusted biased regression estimators are proposed by using an adjustment factor based on signal-to-noise ratio (SNR). The theoretical results are applied to Liu-type estimators using the well known data of Portland Cement Data. The numerical results are in favor of the proposed adjusted estimators in the form of a smaller prediction error sum of squares (PRESS) criterion of the adjusted Liu type estimators compared to the original ones. The adjustment is also applied to the ordinary least squares estimators (OLSE) and other biased estimators such as ordinary ridge regression estimator (ORRE), and Liu estimator (LE). The best results are obtained for OLSE, ORRE, LE, and Liu type(1) estimators. It is concluded that this adjustment can be applied to any significantly regression estimator.

KEYWORDS: Adjusted Estimators, Liu Type Estimators, Ordinary Least Squares Estimator, Ordinary Ridge Regression Estimator, Prediction Error Sum of Squares, Signal to Noise Ratio

1. INTRODUCTION

Several estimators have been proposed to combat the multicollinearity problem. Some of these estimators are the Liu and Liu type estimators due to Liu (1993); and (2003) respectively. Liu estimator proposed by Liu (1993) received a great attention in the literature. (See Akdeniz and Kaciranlar (1995); Kaciranlar et al. (1999); Arslan and Billor (2000); Kaciranlar and Sakalliglu (2001); Torigoe and Ujiie (2006); Rong (2010); and Liu (2011).

Consider the following linear regression model:

$$y = z\gamma + \varepsilon, \tag{1}$$

Where y is an (nx1) vector of standardized response, z is an (nxp) matrix of standardized regressors, γ is a (px1) vector of unknown parameters, and ε is an (nx1) vector of errors such that $\varepsilon \sim N(0, \sigma^2 I)$. Let $\hat{\gamma}$ be the ordinary least squares estimator (OLSE) of γ, defined as,

$$\hat{\gamma} = (z'z)^{-1} z'y, \tag{2}$$

which is the best linear unbiased estimator (BLUE) of γ.

In the case of exiting near multcollinearity among regressors, the characteristic BLUE of OLSE will be of little comfort. The variance of OLSE may be very large, so its accuracy will be reduced. Instead of using OLSE, various biased regression estimators are considered. A popular numerical method to deal with the multicollinearity problem is the ordinary ridge regression estimator (ORRE) proposed by Hoerl and Kennard (1970), which is defined as follows:

$$\hat{\gamma}_k = \left(z'z + kI\right)^{-1} z'y\,, \tag{3}$$

where k is a biasing ridge parameter. The disadvantage of ORRE is that a value of the ridge parameter k may be not large enough to reduce the condition number of the matrix $\left(z'z + kI\right)^{-1}$ when $z'z$ is very ill-conditioned. To overcome this problem, Liu (1993) proposed Liu estimator which is based on the OLSE, $\hat{\gamma}$. As a result, it is found that Liu estimator performs poorly and sometimes gives misleading information. To overcome this problwem, Liu (2003) proposed a new Liu-type estimator which depends on any estimator.

After Liu (2003) introduced his Liu-type estimator, various estimators are proposed based on this estimator. Combining Liu and Liu-type estimators with other biased and unbiased estimators, improving and adjusting Liu and Liue-type estimators are examples for these proposals.(See Kaciranlar et al. (1999); Alheety and Kibria (2009); Li and Yang (2010); Liu (2011); Liu and Gao (2011); Li and Yang (2011); Gruber (2012); Liu et al. (2013), etc...).

The purpose of this paper is to introduce new adjusted Liu-type estimators, and special cases of them. The adjustment based on the idea of signal-to-noise ratio (SNR).

This paper is organized as follows. Section (2) considers Liu type estimators. Section (3) introduces the methodology of the proposed estimators. Section (4) present numerical results based on a simulation study, and a real data based on Cement Portland data. The conclusions of this paper are given in section (5).

2. LIU-TYPE REGRESSION ESTIMATORS

Liu (1993) proposed a new biased estimator as an alternative to the ORRE by combining the Stein (1956) estimator with the ORRE. This estimator is called "Liu estimator" by Akdeniz and Kaciranlar (1995). Liu estimator (LE) is defined as follows:

$$\hat{\gamma}_d = \left(z'z + I\right)^{-1}\left(z'y + d\hat{\gamma}\right), \tag{4}$$

where $\hat{\gamma}$ is the OLSE of γ, and $d \in \left(-\infty, \infty\right)$ is a an arbitrary constant parameter which can used to improve the fit and the statistical properties. The advantage of the Liu estimator over the ORRE, $\hat{\gamma}_k$, in (3) is that Liu estimator, $\hat{\gamma}_d$, is a linear function of d, so it is easy to select an optimal value of d. Akdeniz and Ozturk (2005) derived the distribution density function of the stochastic parameter d by assuming normality.

The LE, $\hat{\gamma}_d$ in (4) is based on OLSE ($\hat{\gamma}$) in (2), so its performance is poor and sometimes gives misleading results. Liu (2003) proposed a new Liu-type estimator (LTE) to overcome the problem of LE as follows,

$$\hat{\gamma}_{k,d} = \left(z'z + kI\right)^{-1}\left(z'y - d\hat{\gamma}^*\right) \tag{5}$$

where k>0 is a biasing parameter which can be used to control the condition number of the matrix, $\left(z'z + kI\right)$;

$d \in (-\infty, \infty)$ is a an arbitrary constant parameter which can used to improve the fit and the statistical properties; and $\hat{\gamma}^*$ can be any estimator of γ.

Liu (2003) considered two choices of $\hat{\gamma}^*$, the first is the OLSE and the second is the ORRE, defined as LTE1 and LTE2, respectively as follows:

LTE1: $\hat{\gamma}_{k,d} = \left[(z'z + kI)^{-1} - d(z'z + kI)^{-1}(z'z)^{-1}\right]z'y$. (6)

LTE2: $\hat{\gamma}_{k,d} = \left[(z'z + kI)^{-1} - d(z'z + kI)^{-2}\right]z'y$. (7)

Sakalliglu and Kaciranlar (2008) proposed a new biased estimator called the k-d class estimator, defined as follows:

$$\hat{\gamma}_{k,d} = (z'z + I)^{-1}(z'y + d\hat{\gamma}_k), \quad (8)$$

where $\hat{\gamma}_k$ is the ORRE of γ. The above k-d class estimator is a special case of Liu-type estimator defined in (5). Sakalliglu and Kaciranlar (2008) compared their estimator in (8) with the OLSE and the two Liu type estimators, LTE1 and LTE2. In this case, The k-d class estimator defined in (8) can be defined as LTE3 as follows:

LTE3: $\hat{\gamma}_{k,d} = \left[(z'z + I)^{-1} + d(z'z + I)^{-1}(z'z + kI)^{-1}\right]z'y$. (9)

Rong (2010) used the above Liu-type estimators LTE1, LTE2, and LTE3 in his proposal of adjusted estimators. He based the adjustment on a general formula for the three estimators and selected the adjusted factor which minimizes the PRESS. In this work, we propose to adjust Liu type estimators by using a different methodology based on a signal to noise ratio (SNR). It is concluded that our adjustment can be applied to any other biased or unbiased estimator.

From the theory and practical point of view, comparing biased estimators is based on the mean squared error (MSE) scalar or matrix criterion (MMSE). (See Sakallioglu et al. (2001); Akdeniz and Erol (2003); Sakalliglu and Kaciranlar (2008)).

All the comparisons, which based on MSE, showed that the best estimator depends on the unknown parameters, the variance of the error term in the linear regression model, and the value of the biasing or shrinkage parameter in biased or shrinkage estimator. Sakalliglu and Kaciranlar (2008) proved that the k-d class estimators has superior properties over the OLSE, ORRE, and the Liu type estimators according to MMSE and MSE.

In this work, it is found that LTE3 is superior over the other estimators according to PRESS criterion.

3. THE METHODOLOGY

Definition

The estimator $\hat{\gamma}_A$ is called an adjusted estimator of the estimator $\hat{\gamma}$, if A is a diagonal matrix such that,

$$\hat{\gamma}_A = A\hat{\gamma}, \quad (10)$$

Where $\hat{\gamma}$ is any estimator of the parameter vector γ, and A=diag(a_{11}, a_{22},…,a_{pp}), such that (a_{11}, a_{22},…,a_{pp})∊R are p scalars.

3.1. The Signal- to-Noise Ratio (SNR)

Any value obtained by a measurement contains two components: the first contains the information of interest, known as the signal, and the other consists of random errors, or noise. The random errors are unwanted because they diminish the accuracy and precision of the measurement.

There have been a large number of definitions of the signal-to-noise ratio (SNR). One of the most important definitions is the one used by Taguchi (1987) in quality engineering. Taguchi (1987) introduced the following SNR for evaluating the performance of the linear regression model in (1), as follows:

$$SNR = \frac{\gamma^2}{\sigma_\circ^2}, \tag{11}$$

where γ is an unknown regression parameter, and $\sigma_\circ$ is the standard deviation of the noise or the error term of the model.

An alternative definition of SNR is the reciprocal of the coefficient of variation, i.e., the ratio of mean to standard deviation of a signal or measurement as follows:

$$SNR = \frac{\mu}{\sigma}, \tag{12}$$

where μ is the signal mean or expected value and σ is the standard deviation of the noise or an estimator.

3.2. An Algorithm for the Proposed Adjusted Estimators

SNR is similar to testing whether of γ, in the linear regression model in (1), is significantly different from zero which can be defined as follows:

$$SNR = \frac{\hat{\gamma}}{\hat{\sigma}_{\hat{\gamma}}} = t(\hat{\gamma}), \tag{13}$$

where $t(\hat{\gamma})$ is a t-statistic of testing the significance of γ, and $\hat{\sigma}_{\hat{\gamma}}$ is the standard error of $\hat{\gamma}$. Thus SNR is considered large, that is representing a signal if, for example, SNR > 3 for a confidence level of 99.9%. This implies an existence of a signal over and above noise. It can be concluded that:

$$SNR = \frac{\hat{\gamma}}{\hat{\sigma}_{\hat{\gamma}}} = t(\hat{\gamma}) > 3. \tag{14}$$

Then number three can be used as a bench mark, such that SNR should not be less than three. If SNR is less than 3, then the data implies some noise.

From Eq.(12), the estimator $\hat{\gamma}$ can be defined as follows:

$$\hat{\gamma} = \hat{\sigma}_{\hat{\gamma}} \cdot t(\hat{\gamma}) = 3\hat{\sigma}_{\hat{\gamma}} . \tag{15}$$

An adjusted estimator of $\hat{\gamma}$ can be obtained as follows:

$$\hat{\gamma}_A = A \cdot \hat{\gamma} = 3A\hat{\sigma}_{\hat{\gamma}_A} , \tag{16}$$

where A= diag(a_1,a_2,…..,a_p) is a diagonal matrix defined as an adjusted factor of $\hat{\gamma}$, and a_1,a_2,…..,$a_p \in$ R are scalars. The adjusted factor A can be found as:

$$A = {3.\hat{\sigma}_{\hat{\gamma}_A}} \big/ {\hat{\gamma}_A} . \tag{17}$$

The following simple iterative algorithm will be used in finding the adjusted estimator $\hat{\gamma}_A$:

- Find the initial adjusted factor $A_\circ$: $A_\circ = 3\hat{\sigma}_{\hat{\gamma}_\circ} / \hat{\gamma}_\circ$, find the adjusted parameter $\hat{\gamma}_1$: $\hat{\gamma}_1 = A_\circ \hat{\gamma}_\circ = 3\hat{\sigma}_{\hat{\gamma}_\circ}$, and find

  $$D_1 = \left\| \hat{\gamma}_1 - \hat{\gamma}_\circ \right\|^2 ,$$

  where $\hat{\sigma}_{\hat{\gamma}_\circ}$ is the standard error of $\hat{\gamma}_\circ$, $\hat{\gamma}_\circ$ is the initial estimated parameter vector before adjustment, and $\left\| \cdot \right\|^2$ denotes the squared norm.

- Find the first adjusted factor A_1: $A_1 = 3\hat{\sigma}_{\hat{\gamma}_1} / \hat{\gamma}_1$, find the adjusted parameter $\hat{\gamma}_2$: $\hat{\gamma}_2 = A_1 \hat{\gamma}_1 = 3\hat{\sigma}_{\hat{\gamma}_1}$, and find

  $$D_2 = \left\| \hat{\gamma}_2 - \hat{\gamma}_1 \right\|^2 ,$$

  where $\hat{\sigma}_{\hat{\gamma}_1}$ is the standard error of $\hat{\gamma}_1$, and $\hat{\gamma}_1$ is the estimated parameter vector as defined in step (1).

- Find the second adjusted factor A_2: $A_2 = 3\hat{\sigma}_{\hat{\gamma}_2} / \hat{\gamma}_2$,

  find the adjusted parameter $\hat{\gamma}_3$: $\hat{\gamma}_3 = A_2 \hat{\gamma}_2 = 3\hat{\sigma}_{\hat{\gamma}_2}$, and find

  $$D_3 = \left\| \hat{\gamma}_3 - \hat{\gamma}_2 \right\|^2 ,$$

  where $\hat{\sigma}_{\gamma_2}$ is the standard error of $\hat{\gamma}_2$, and $\hat{\gamma}_2$ is the estimated parameter vector as defined in step (2).

- Repeat Finding the adjusted factor and the adjusted parameter estimator m times A_m, and $\hat{\gamma}_m$, respectively. Also, the squared norm, D_m as follows:

$A_m = 3\hat{\sigma}_{\gamma_m} / \hat{\gamma}_m$, and $\hat{\gamma}_m = A_m \hat{\gamma}_m = 3\hat{\sigma}_{\gamma_m}$,

Such that $D_m = \left\| \hat{\gamma}_m - \hat{\gamma}_{m-1} \right\|^2 \approx 0$

where $\hat{\sigma}_{\hat{\gamma}_m}$ is the standard error of $\hat{\gamma}_m$, and $\hat{\gamma}_m$ is the estimated parameter vector as defined in step (m).

- Finally take $\hat{\gamma}_A = \hat{\gamma}_m$ as the adjusted estimated parameter vector.

The adjustment will be applied to the Liu type estimators and also to their special cases, OLSE and ORRE for comparison.

3.3. The Prediction Error Sum of Squares (PRESS)

The prediction error sum of squares (PRESS) statistic, proposed by Allen (1974), is used to compare different models. The PRESS statistic does not depend on some particular model parameters, but on the model itself. The prediction error sum of squares (PRESS) statistic is a form of cross-validation used in regression analysis to provide a summary measure of the fit of a model to a sample of observations that were not themselves used to estimate the model. (See Allen (1971; 1974)).

In this paper PRESS is used to compare the different proposed estimators with other unbiased and biased estimators as will be shown in the following section. PRESS is simply calculated as the sums of squares of the prediction residuals for those observations as follows:

$$PRESS = \sum_{i=1}^{n} \left(y_i - \hat{y}_{i,-i} \right)^2 = \sum_{i=1}^{n} \left(y_i - z_i' \hat{\gamma}_{-i} \right)^2 , \tag{18}$$

where $\hat{y}_{i,-i}$ is denotes the fitted value i without the ith observation, and $\hat{\gamma}_{i,-i}$ is any estimator of γ_i after discarding the ith observation.

The following estimators will be considered in this work:

- The OLSE: $\hat{\gamma}_{-i}^{\text{(OLSE)}} = \left(Z'Z - z_i z_i' \right)^{-1} \left(Z'y - z_i y_i \right)$
- The ORRE: $\hat{\gamma}_{-i}^{\text{(ORRE)}} = \left(Z'Z + kI - z_i z_i' \right)^{-1} \left(Z'y - z_i y_i \right)$
- The LTE1: $\hat{\gamma}_{-i}^{\text{(LTE1)}} = \left[\left(Z'Z + kI - z_i z_i' \right)^{-1} - d\left(Z'Z + kI - z_i z_i' \right)^{-1} \left(Z'Z - z_i' z_i \right)^{-1} \right] \left(Z'y - z_i y_i \right)$
- The LTE2: $\hat{\gamma}_{-i}^{\text{(LTE2)}} = \left[\left(Z'Z + kI - z_i z_i' \right)^{-1} - d\left(Z'Z + kI - z_i z_i' \right)^{-2} \right] \left(Z'y - z_i y_i \right)$
- The LTE3: $\hat{\gamma}_{-i}^{\text{(LTE3)}} = \left[\left(Z'Z + I - z_i z_i' \right)^{-1} + d\left(Z'Z + I - z_i z_i' \right)^{-1} \left(Z'Z + kI - z_i z_i' \right)^{-1} \right] \left(Z'y - z_i y_i \right)$

Also, the adjusted versions of these estimators will be considered as follows:

- The adjusted OLSE: $\hat{\gamma}_{-i\,\text{(AOLSE)}} = A_1 \left(Z'Z - z_i z_i' \right)^{-1} \left(Z'y - z_i y_i \right)$

- The adjusted ORRE: $\hat{\gamma}_{-i\,(\text{AORRE})} = A_2 \left(Z'Z + kI - z_i z_i'\right)^{-1}\left(Z'y - z_i y_i\right)$
- The adjusted LTE1: $\hat{\gamma}_{-i}^{\,(\text{ALTE1})} = A_3 \left[\left(Z'Z + kI - z_i z_i'\right)^{-1} - d\left(Z'Z + kI - z_i z_i'\right)^{-1}\left(Z'Z - z_i' z_i\right)^{-1}\right]\left(Z'y - z_i y_i\right)$
- The adjusted LTE2: $\hat{\gamma}_{-i\,(\text{ALTE2})} = A_4 \left[\left(Z'Z + kI - z_i z_i'\right)^{-1} - d\left(Z'Z + kI - z_i z_i'\right)^{-2}\right]\left(Z'y - z_i y_i\right)$
- The adjusted LTE3: $\hat{\gamma}_{-i\,(\text{ALTE3})} = A_5 \left[\left(Z'Z + I - z_i z_i'\right)^{-1} + d\left(Z'Z + I - z_i z_i'\right)^{-1}\left(Z'Z + kI - z_i z_i'\right)^{-1}\right]\left(Z'y - z_i y_i\right)$

The performance of these estimators will be shown and compared according the PRESS for the estimator.

4. NUMERICAL RESULTS

To investigate our proposed estimators discussed in this paper, the well-known dataset on Portland cement due to Woods et al (1932) is used. These data come from an experiment investigation of the heat evolved during the setting and hardening of Portland cements of varied composition, and the dependence of this heat on the percentages of four components in the clinkers from which the cement was produced. A data frame with 13 observations on the following 5 variables:

X_1: Tricalcium Aluminate.

X_2: Tricalcium Silicate.

X_3: Tetracalcium Aluminoferrite.

X_4: Dicalcium Silicate.

It is found that the condition number of the matrix X is about 6051.419, which means that the design matrix is ill-conditioned, and the OLSE is no longer a good estimator using the MSE criterion. The theoretical results of this paper are well supported by this dataset as will be shown in the following sections.

In this work, the estimators will be compared according to PRESS, the best estimator is the one which has smaller value of PRESS. Also, the relative improvement (RI) of the adjusted estimator compared $(A\hat{\gamma})$ to the original estimator $(\hat{\gamma})$ will be computed as follows:

$$RI(\hat{\gamma}, A\hat{\gamma}) = \frac{PRESS \text{ of } \hat{\gamma} - PRESS \text{ of } A\hat{\gamma}}{PRESS \text{ of } \hat{\gamma}}$$

In this paper, it is shown that the optimal biasing factors are d= -0.1 and k=0.2. The repetition of the adjustment is stopped for all biased estimator at m=3 except for OLSE at m=1. In Table (1), and Table (2) it is found that the proposed adjustment factor for the OLSE is (0.253488, 0.12470, 0.63085, 0.255449) and the PRESS before adjustment is 98.5491 but is equal 17.00863 after using the adjustment factor with a higher relative improvement of 82.74%. (See Table (3)).

The proposed adjustment factor for the ORRE is (0.25339, 0.124523, 0.631725, 0.255188) and the PRESS before adjustment is 98.4282 but is equal 16.96145 after using the adjustment fact with high relative improvement of 73.62%. (See Table (3)). The proposed adjustment factor for the Liu estimator is (0.252766, 0.123185, 0.63443, 0.252968) and the PRESS before adjustment is 98.4572 but is equal 16.9325 after using the adjustment factor with high relative improvement of 82.70%.(See Table (3)). The proposed factor adjustment for the LTE1 is (0.253365, 0.124567, 0.631287, 0.255293) and the PRESS before adjustment is 98.5491 but is equal 16.99768 after using the adjustment factor with a relative improvement of 82.74%(See Table (3)). The proposed adjustment factor for the LTE2 is (0.857276, 0.873291, 1.121333, 1.18959) and the PRESS before adjustment is 88.30704 but is equal 54.8621 after using the adjustment factor with a relative improvement of 37.94%. (See Table (3)). The proposed adjustment factor for the LTE3 is (0.849331, 0.872642, 1.122145, 1.189543) and the PRESS before adjustment is 89.58145 but is equal 54.79164 after using the adjustment factor with a higher relative improvement of 38.84%.(See Table (3)). These results are obtained using R language and "lrmest" package and Mini Tab.

Table 1: Parameter Estimates, Adjusted Parameter Estimates, PRESS Values for the Ordinary Least Squares Estimators (OLSE) Using Portland Cement Data (PCD) Due to Woods et al. (1932)

Parameter Estimates	OLSE	Adjusted OLSE	ORRE	Adjusted ORRE	LE	Adjusted LE
$\hat{\gamma}_1$	2.1930	0.5559	2.1903	0.5550	2.1779	0.5505
$\hat{\gamma}_2$	1.15333	0.14382	1.1540	0.1437	1.5680	0.1425
$\hat{\gamma}_3$	0.7585	0.4785	0.7565	0.4779	0.7476	0.4743
$\hat{\gamma}_4$	0.48632	0.12423	0.4867	0.1242	0.4886	0.1236
PRESS	98.5491	17.00863	98.4282	16.96145	97.8528	16.9325

Table 2: Parameter Estimates, Adjusted Parameter Estimates, PRESS Values for the Three Liu Type Estimators, LTE1, LTE2, and LTE3 Using Portland Cement Data (PCD) Due to Woods et al. (1932)

Parameter Estimates	LTE1	Adjusted LTE1	LTE2	Adjusted LTE2	LTE3	Adjusted LTE3
$\hat{\gamma}_1$	2.1917	0.5553	2.0204	1.73204	2.0409	1.7334
$\hat{\gamma}_2$	1.1536	0.1437	1.7118	1.49490	1.7282	1.5081
$\hat{\gamma}_3$	0.7575	0.4782	0.9000	1.00920	0.9063	1.0170
$\hat{\gamma}_4$	0.4865	0.1242	0.8703	1.03530	0.8779	1.0443
PRESS	98.4926	16.99768	88.30704	54.80621	89.58145	54.79164

Table 3: The Relative Improvement (RI) of PRESS Values for the Different Estimators before and after Adjustment, OLSE, ORRE, LE, and the Three Liu Type Estimators, LTE1, LTE2, and LTE3 Using Portland Cement Data (PCD) Due to Woods et al. (1932)

PRESS for Different Estimators	OLSE	ORRE	LE	LTE1	LTE2	LTE3
PRESS before adjustment	98.5491	98.4282	97.8528	98.4926	88.30704	89.58145
PRESS after adjustment	17.00863	16.96145	16.9325	16.99768	54.80621	54.79164
RI	82.74%	73.62%	82.70%	82.74%	37.94%	38.84%

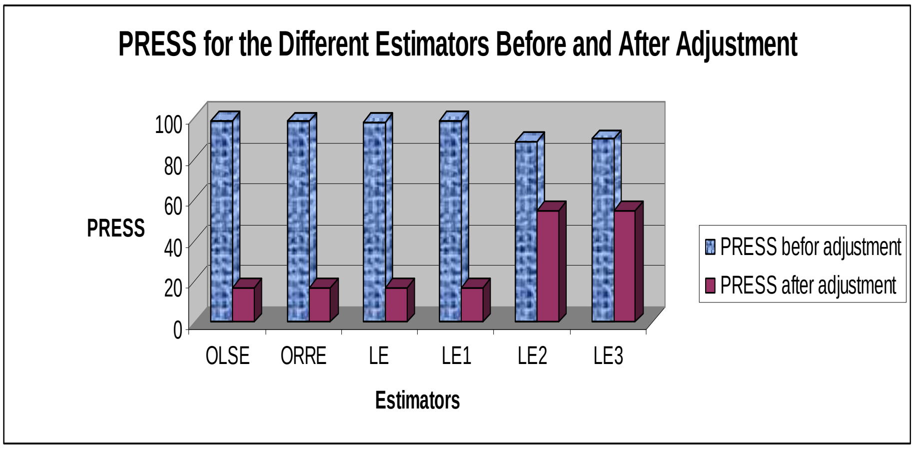

Figure 1: PRESS for Different Estimators (OLSE, ORRE, LE, LTE1, LTE2, and LTE3) Before and after Adjustment

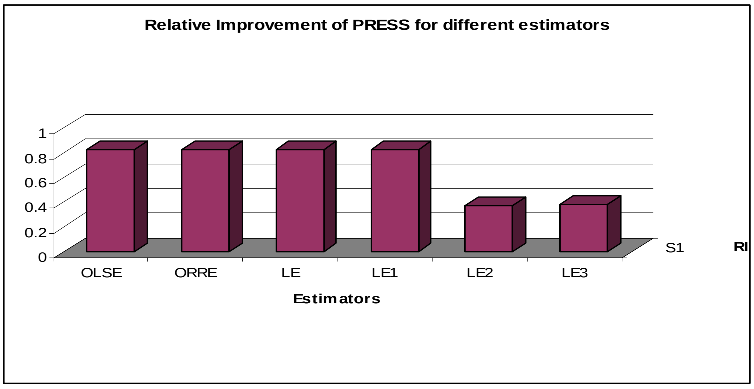

Figure 2: Relative Improvement of PRESS for Different Estimators (OLSE, ORRE, LE, LTE1, LTE2, and LTE3)

5. CONCLUSIONS

In this paper, we propose new adjusted biased regression estimators by using an adjustment factor based on signal-to-noise ratio (SNR) and an iterative algorithm. The theoretical results are applied to the most recently biased estimators that are Liu-type estimators using the well known multicollinear data of Portland Cement Data due to Wood et al. (1932). The numerical results are in favor of our proposed adjusted estimators in the form of a smaller prediction error sum of squares (PRESS) criterion of the adjusted Liu type estimators compared to the original ones. The adjustment is also applied to the ordinary least squares estimators (OLSE) and other biased estimators such as ordinary ridge regression estimator (ORRE), and Liu estimator (LE). The best results are obtained for OLSE, ORRE, LE, and LET1 estimators in the form of large relative improvement of the adjusted estimator compared to the original estimator. It can be concluded that this adjustment can be applied to any significantly regression estimator. More work is needed in the area of adjusted regression estimators using different methodology.

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